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# Conversion method of cap vols across tenors
## The case of the 12M Euribor index

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This article describes a simple procedure for approximating the cap volatility corresponding to the 12M Euribor tenor.

We compare the resulting expression in the single- and dual-curve worlds.

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# Introduction

Underlying liabilities can often be linked to floating mortgages or other floating loans, the rate of which is adjusted annually. Risk managers and traders that wish to value or hedge such liabilities are in principle required to value a position on a 12M underlying index rate. Unfortunately there is no liquid quoted data for such an index on most of the data providers, including Bloomberg. This brings in a necessity to invent a methodology for the construction of an arbitrage-free way to extrapolate the 12M cap vols from another tenor.

In this article we describe a simple conversion method that takes as input the 6M EUR cap vols and outputs the 12M cap vols. The method is exact under the single-curve assumption. We provide some justification for its validity under the dual-curve assumption. Finally, some comparisons are given versus Bloomberg's Swap Manager.

Although the presented case study refers to the Euribor 12M, the methodology can be easily translated to other currencies and underlyings.

# Setting the scene

As we are interested in the valuation of caps, floors and swaptions it is appropriate to fix the notation and some ideas that will be used in the next sections.

A cap is a series of caplets, each of which pays the positive part of the difference between the forward rate between the start and end dates of the caplet and the strike. The forward rate is fixed at the start date of the caplet period.
A (payer) swaption is the option to enter into a swap. The swaption is characterised by (i) the maturity which is the end of the option and, also, the start of the swap and (ii) the tenor which is the period of the swap.

In the table below we give the defining relations of the discounted cap and swaption prices:

| **Discounted MTM at valuation date** |
|---|
| $Cap(T_0) = = \mathrm{E}\left[ \sum_{i=s+1}^{e} D(T_0, T_i) \cdot 2_{i-1,i} \cdot \left(F(T_{i-1}, T_{i-1}, T_i) - K\right)^+ \middle\vert F_{T_0} \right]$ |
| $Swaption(T_0) = = \mathrm{E}\left[ D(T_0, T_s) \cdot \left( \sum_{i=s+1}^{e} P(T_s, T_i) \cdot 2_{i-1,i} \cdot F(T_s, T_{i-1}, T_i,) - \sum_{i=s+1}^{e} P(T_s, T_i) \cdot 2_{i-1,i} \cdot K \right)^+ \middle\vert F_{T_0} \right]$ |

where we have used the abbreviation $(x)^+ = \max(x,0)$ and

- $T_s$ denotes the start date (for a cap this is the start of the payment schedule, for a swaption it is the start of the swap)
- $T_e$ is the end date.
- $i$ is an index running over the various time-points of interest: $T_s, T_{s+1}, \ldots, T_e$
- $2_{i-1,i}$ is the yearfraction between $T_{i-1}$ and $T_i$
- $F(T_0, T_{i-1}, T_i,)$ is the forward rate between dates $T_{i-1}$ and $T_i$ as observed at time $T_0$.
- $K$ is the strike of the deal.
- $D(T_0, T_i)$ represents the stochastic discount factor between the two dates.
- $P(T_0, T_i)$ represents the zero-coupon bond between the two dates. This is obtained from the yield curve.
- The notation $\mathrm{E}\left[\cdots \middle\vert F_T\right]$ denotes a risk-neutral expectation with filtration up to time $T$.

At the valuation date, $T_0$, stochasticity enters into the valuation of the cap through the discount factor and the forward rate, both of which are in general unknown.
The undiscounted payoff of a swaption, however, is expressed in terms of zero-coupon bonds. Therefore stochasticity enters only through the forward rate.

It is market practice to price these two vanilla instruments using either Black-Scholes of Bachelier formulas.

Let us focus on the Bachelier representation, which assumes that the absolute increments of the underlying (the forward rate) are normally distributed random variables of zero drift and volatility ± .

The discounted prices of caps and swaptions under this model are given by:

| **Price at** $T_0$ | |
|---|---|
| **Cap (A1)** | $Cap(T_0) = \\ = \sum_{i=s+1}^{e} P(T_0,T_i) \cdot 2_{\,i-1,i} \cdot Bachelier\left(F(T_0,T_{i-1},T_i), K, \pm_i, {}^2(T_0,T_{i-1})\right)$ |
| **Swaption (A2)** | $Swaption(T_0) = \\ = \left(\sum_{i=s+1}^{e} P(T_0,T_i) \cdot 2_{\,i-1,i}\right) \cdot Bachelier\left(R(T_0,T_{i-1},T_i), K, \pm_i, {}^2(T_0,T_{i-1})\right)$ |

where the Bachelier formula is:

$$Bachelier(F,K,\pm_i,{}^2) = (F-K)\cdot N\left(\frac{F-K}{\pm\sqrt{2}}\right) + \pm\sqrt{2}\cdot n\left(\frac{F-K}{\pm\sqrt{2}}\right)$$

and, as usual, $N(.)$ and $n(.)$ represent the normal cumulative and normal distribution density function respectively.

For the valuation of the above two instruments, we note that the cap requires as input the forward rate $F(T_0,T_{i-1},T_i)$ while the swaption requires the forward swap rate $R(T_0,T_{i-1},T_i)$. Also note that the cap effectively discounts each caplet at the zero-coupon bond rate, while the swaption effectively discounts using the annuity.

The story is thus far only half-told regarding the inputs to the above formulas as we have neglected one of the key considerations: The single-curve vs dual-curve assumption.

Under the single-curve assumption the zero-coupon bonds are linked to the forward rates and forward swap rates through the formula:

| **Single-curve assumption** | |
|---|---|
| **Forward rate (B1)** | $F(T_i,T_j,T_k) = \frac{1}{2(T_j,T_k)} \cdot \left(\frac{P(T_i,T_j)}{P(T_i,T_k)} - 1\right)$ |
| **Forward swap rate (B2)** | $R(T_i,T_j,T_k) = \frac{P(T_i,T_j) - P(T_i,T_k)}{\sum_{m=j+1}^{k} 2(T_{m-1},T_m) \cdot P(T_i,T_m)}$ |

On the contrary, under the dual-curve assumption, which is the market practice, the relation involves various curves.

| **Dual-curve assumption** | |
|---|---|
| **x-Month Forward rate (C1)** | $F_{xM}\left(T_i, T_j, T_k\right) = \frac{1}{2\left(T_j, T_k\right)} \cdot \left( \frac{P_{xM}\left(T_i, T_j\right)}{P_{xM}\left(T_i, T_k\right)} - 1 \right)$ |
| **x-Month Forward swap rate (C2)** | $R_{xM}\left(T_i, T_j, T_k\right) = \frac{\sum_{m=j+1}^{k} F_{xM}\left(T_i, T_{m-1}, T_m\right) \cdot 2\left(T_{m-1}, T_m\right) \cdot P_{OIS}\left(T_i, T_m\right)}{\sum_{m=j+1}^{k} 2\left(T_{m-1}, T_m\right) \cdot P_{OIS}\left(T_i, T_m\right)}$ |

where $P_{OIS}\left(T_i, T_j\right)$ and $P_{xM}\left(T_i, T_j\right)$ correspond respectively to the discount and forward x-Month curves.

# Bachelier formula’s translation property

The Bachelier formula is one of the two common ways to quote a price. It is typically used in conjunction with a stochastic model, such as the SABR model: The stochastic model provides a volatility while the Bachelier formula is used as a quoting device to convert the volatility into a price.

One of the advantages of the Bachelier formula is that a negative strike does not lead to unpleasant divergences under the negative-rate environment, unlike the Black formula. For this reason, there are volatility quotes available for negative strikes. On the contrary, under the Black formula one has to apply a shift to avoid divergences.

The Bachelier formula also entails an interesting translation-invariance property with respect to the forward and the strike, namely:

$$\begin{aligned} Bachelier\left(F, K+f, \pm, {}^2\right) &= \left(F-K-f\right)\cdot N\left(\frac{F-K-f}{\pm\sqrt{{}^2}}\right) + \pm\sqrt{{}^2}\cdot n\left(\frac{F-K-f}{\pm\sqrt{{}^2}}\right) \\ &= Bachelier\left(F-f, K, \pm, {}^2\right) \end{aligned}$$

This property will be used in the next section to convert an adjustment to the strike into an adjustment to the forward swap rate.

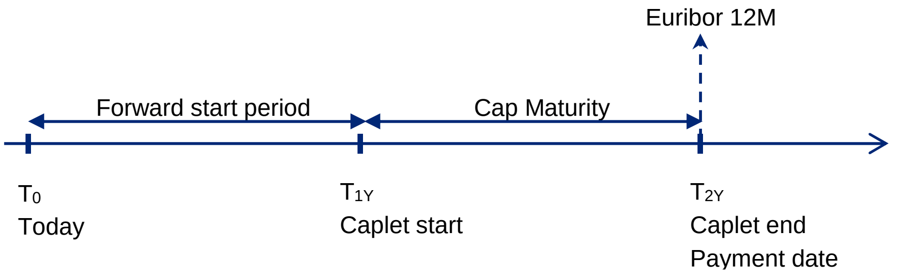
# An exact relation for the 12M cap vols in the single-curve world

There are no quotations on Bloomberg regarding the EUR 12M cap volatility. This means that if one wants to price an instrument on the 12M Euribor, then this underlying needs to be constructed from another tenor.

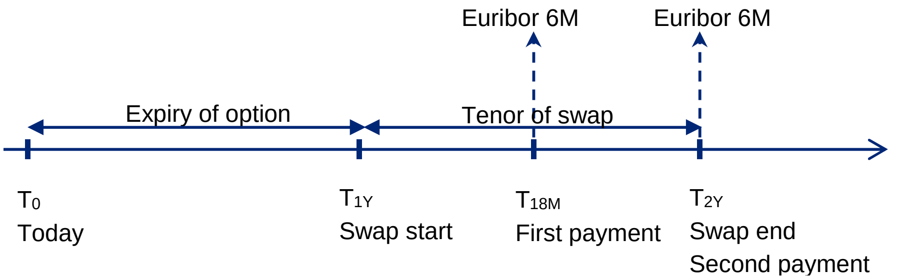
Since the 6M tenor is the most liquid underlying on the EUR currency it may be a good idea to extrapolate the 12M vols from the 6M vols.

In order to do so one argues in the following way:

A caplet that starts in 1Y with underlying 12M Euribor is essentially an option on a swap that pays the 12M Euribor over the period between 1Y and 2Y with one payment at 2Y (see figure below).




A swaption with underlying 6M Euribor, with tenor 1Y and expiry 1Y is an option on a swap that pays 6M Euribor twice (see figure below).




In this setting and under the single-curve assumption it can be demonstrated that one payment of 12M Euribor at 2Y equals two semi-annual payments of 6M Euribor (one at 1.5Y and the second at 2Y). This equivalence in the two payoffs is exact in a single-curve world.

In order to see this we write out the payoffs of the cap at $T_{1Y}$ which is the start date of the caplet (fixing of the forward rate) and also the start date of the swap.

We are interested to compare the price at $T_{1Y}$ of a cap starting at that date with underlying the 12M index versus the price of a swaption on a 6M index with tenor 1Y.

Taking an expectation in the expression of the payoff a cap (with filtration up to $T_{1Y}$ ) allows us to separate the deterministic part (involving the forward rate which has been fixed at $T_{1Y}$ ) from the stochastic part (the discount factor). The latter is then equivalent to the zero-coupon bond.

The price of a cap at 1Y starting in 1Y with underlying Euribor 12M has one payment and is given by

$$Cap(T_{1Y}) = P(T_{1Y}, T_{2Y}) \cdot 2_{1Y,2Y} \cdot \left(F(T_{1Y}, T_{1Y}, T_{2Y}) - K\right)^{+}$$

Inserting the single-curve assumption (B1) leads to

$$Cap(T_{1Y}) = P(T_{1Y}, T_{2Y}) \cdot 2_{1Y,2Y} \cdot \left(\frac{1}{2_{1Y,2Y}} \cdot \left(\frac{P(T_{1Y}, T_{1Y})}{P(T_{1Y}, T_{2Y})} - 1\right) - K\right)^{+}$$
$$= \left(1 - P(T_{1Y}, T_{2Y}) - K \cdot P(T_{1Y}, T_{2Y})\right)^{+}$$

On the other hand, a swaption on the 6M Euribor index is composed of a floating part and an annuity part. The floating part pays semi-annually (thus 2 payments) while the annuity part pays annually (thus 1 payment). This implies that

$$Swaption(T_{1Y}) =$$
$$= \begin{pmatrix} P(T_{1Y}, T_{18M}) \cdot 2_{1Y,18M} \cdot F(T_{1Y}, T_{1Y}, T_{18M}) + P(T_{1Y}, T_{2Y}) \cdot 2_{18M,2Y} \cdot F(T_{1Y}, T_{18M}, T_{2Y}) \\ - K \cdot 2_{1Y,18M} \cdot P(T_{1Y}, T_{2Y}) \end{pmatrix}^{+}$$

Inserting the single-curve assumption (B1) leads to

$$Swaption(T_{1Y}) =$$
$$= \begin{pmatrix} P(T_{1Y}, T_{18M}) \cdot 2_{1Y,18M} \cdot \dfrac{1}{2_{1Y,18M}} \cdot \left(\dfrac{P(T_{1Y}, T_{1Y})}{P(T_{1Y}, T_{18M})} - 1\right) \\ + P(T_{1Y}, T_{2Y}) \cdot 2_{18M,2Y} \cdot \dfrac{1}{2_{18M,2Y}} \cdot \left(\dfrac{P(T_{1Y}, T_{18M})}{P(T_{18M}, T_{2Y})} - 1\right) \\ - K \cdot 2_{1Y,2Y} \cdot P(T_{1Y,2Y}) \\ \end{pmatrix}^{+}$$
$$= \left(1 - P(T_{1Y,2Y}) - K \cdot P(T_{1Y}, T_{2Y})\right)^{+}$$

We notice that the price of a swaption with tenor 1Y and underlying the 6M index equals that of a forward-starting cap with underlying the 12M index and maturity 1Y:

$$Swaption_{6M}(T_{1Y,} K) = Cap_{12M}(T_{1Y}, K)$$

This exact relation between the market price of a 6M swaption vs that of a 12 cap cannot be extended to an exact relation between the 6M swaption vol vs the 12 cap vol. The reason is that the two vols are representations of two slightly different formulas; the formula for swaptions involves the forward swap rate as underlying and discounts by the annuity while the formula for a caplet involves the forward rate as underlying and discounts by the OIS curve. However, in circumstances where the two underlyings and discount rates are similar, approximating 12M cap vols by 6M swaption vols can be reasonable.

# A proxy for the 12M cap vols in the dual-curve world

The single-curve assumption is a crucial element to prove the identity of the two payoffs. However, we know that this assumption is incorrect. To this end, let us introduce the spread between OIS and the 6M zero-coupon curve as seen at time $T_{\sim}$ :

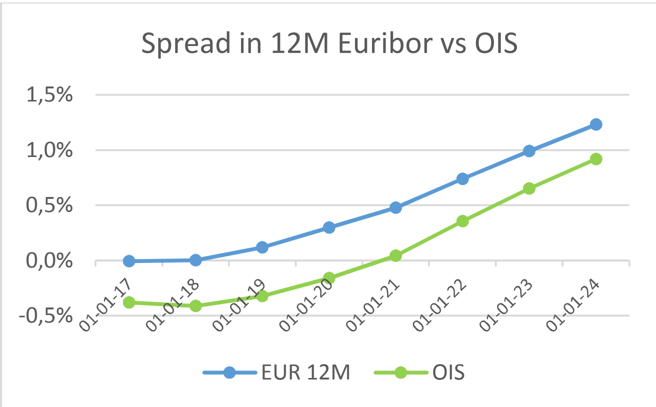
$$S_{6M}(T_{\sim}) = F_{6M}(T_{\sim}, T\ ) - F_{OIS}(T_{\sim}, T\ , T\ + 6M)$$

where our notation $F_{2}(T_{\sim}, T\ ) = F(T_{\sim}, T\ , T\ + {}^{2})$ indicates a forward rate as seen from time $T_{\sim}$ for a period starting at $T$ and finishing at $T\ + {}^{2}$ . Similarly the spread between OIS and the12M zero-coupon curve:

$$S_{12M}(T_{\sim}) = F_{12M}(T_{\sim}, T\ ) - F_{OIS}(T_{\sim}, T\ , T\ + 12M)$$

We are assuming that the spread does not carry a term-structure across maturities and it depends only on the value-date as the viewpoint of the computation (i.e. in the formula above the left-hand side has no dependency on $T$ ). This is justifiable by inspection of the market quotes of the spread.

Indicatively, the figure below shows the spread between the 12M Euribor and OIS rate. The difference between the two curves does not vary significantly, thereby supporting the assumption.




Under the dual-curve world the payoff of the 6M swaption valued at $T_{1Y}$ will be:

$$Swaption_{6M}(T_{1Y}) = \left( \begin{array}{l} P_{OIS}(T_{1Y},T_{18M})\cdot{}^{2}{}_{1Y,18M}\cdot F_{6M}(T_{1Y},T_{1Y}) \\ + P_{OIS}(T_{1Y},T_{2Y})\cdot{}^{2}{}_{18M,2Y}\cdot F_{6M}(T_{1Y},T_{18M}) \\ - K\cdot{}^{2}{}_{1Y,18M}\cdot P_{OIS}(T_{1Y},T_{2Y}) \end{array}\right)^{+}$$

Expressing now the swaption in terms of the OIS-6M spread gives

$$
\begin{aligned}
& Swaption_{6M}\left(T_{1Y}\right)= \\
& =\begin{pmatrix}
P_{OIS}\left(T_{1Y},T_{18M}\right)\cdot 2_{1Y,18M}\cdot\left(F_{OIS}\left(T_{1Y},T_{1Y},T_{1Y}+6M\right)+S_{6M}\left(T_{1Y}\right)\right) \\
+P_{OIS}\left(T_{1Y},T_{2Y}\right)\cdot 2_{18M,2Y}\cdot\left(F_{OIS}\left(T_{1Y},T_{18M},T_{18M}+6M\right)+S_{6M}\left(T_{1Y}\right)\right) \\
-P_{OIS}\left(T_{1Y},T_{2Y}\right)\cdot 2_{1Y,2Y}\cdot K
\end{pmatrix}^{+}
\end{aligned}
$$

Pulling together the spreads and using the identity

$$
\begin{aligned}
& 2_{1Y,18M}\cdot P_{OIS}\left(T_{1Y},T_{18M}\right)F_{OIS}\left(T_{1Y},T_{1Y},T_{18M}\right)+2_{18M,2Y}\cdot P_{OIS}\left(T_{1Y},T_{2Y}\right)F_{OIS}\left(T_{1Y},T_{18M},T_{2Y}\right)= \\
& =1-P_{OIS}\left(T_{1Y},T_{2Y}\right)=2_{1Y,2Y}\cdot P_{OIS}\left(T_{1Y},T_{2Y}\right)F_{OIS}\left(T_{1Y},T_{1Y},T_{2Y}\right)
\end{aligned}
$$

leads to

$$
\begin{aligned}
& Swaption_{6M}\left(T_{1Y}\right)= \\
& =\begin{pmatrix}
S_{6M}\left(T_{1Y}\right)\cdot\left(P_{OIS}\left(T_{1Y},T_{2Y}\right)\cdot 2_{18M,2Y}+P_{OIS}\left(T_{1Y},T_{18M}\right)\cdot 2_{1Y,18M}\right) \\
+2_{1Y,2Y}\cdot P_{OIS}\left(T_{1Y},T_{2Y}\right)\cdot F_{OIS}\left(T_{1Y},T_{1Y},T_{2Y}\right) \\
-2_{1Y,2Y}\cdot P_{OIS}\left(T_{1Y},T_{2Y}\right)\cdot K
\end{pmatrix}^{+}
\end{aligned}
$$

In the second line of the right-hand side we now inject the 12M spread:

$$
Swaption_{6M}\left(T_{1Y}\right)=\begin{pmatrix}
P_{OIS}\left(T_{1Y},T_{2Y}\right)\cdot 2_{1Y,2Y}\left(F_{12M}\left(T_{1Y},T_{1Y}\right)-S_{12M}\left(T_{1Y}\right)\right) \\
-P_{OIS}\left(T_{1Y},T_{2Y}\right)\cdot 2_{1Y,2Y}\cdot \\
\left(K-\dfrac{S_{6M}\left(T_{1Y}\right)}{2_{1Y,2Y}}\cdot\left(2_{18M,2Y}+2_{1Y,18M}\dfrac{P_{OIS}\left(T_{1Y},T_{18M}\right)}{P_{OIS}\left(T_{1Y},T_{2Y}\right)}\right)\right)
\end{pmatrix}^{+}
$$

Switching from OIS zero-coupons to OIS forward rates results in

$$
\begin{aligned}
& Swaption_{6M}\left(T_{1Y}\right)= \\
& =P_{OIS}\left(T_{1Y},T_{2Y}\right)\cdot 2_{1Y,2Y} \\
& \cdot\left(F_{12M}\left(T_{1Y},T_{1Y}\right)-\left(K+S_{12M}\left(T_{1Y}\right)-\frac{S_{6M}\left(T_{1Y}\right)}{2_{1Y,2Y}}\cdot\left(2_{1Y,2Y}+2^{2}_{1Y,18M}F_{OIS}\left(T_{1Y},T_{18M},T_{2Y}\right)\right)\right)\right)^{+} \\
& =P_{OIS}\left(T_{1Y},T_{2Y}\right)\cdot 2_{1Y,2Y} \\
& \cdot\left(F_{12M}\left(T_{1Y},T_{1Y}\right)-\left(K+S_{12M}\left(T_{1Y}\right)-S_{6M}\left(T_{1Y}\right)\cdot\left(1+\frac{2^{2}_{1Y,18M}}{2_{1Y,2Y}}\cdot F_{OIS}\left(T_{1Y},T_{18M},T_{2Y}\right)\right)\right)\right)
\end{aligned}
$$

We can further simplify this expression by arguing that $\left|S_{6M}(T_{1Y}) \cdot F_{OIS}(T_{1Y}, T_{18M}, T_{2Y})\right| << \left|S_{12M}(T_{1Y})\right|$. This is justified by the fact that the forward rate is typically a quantity much less than 1 and furthermore the 6M spread is smaller than the 12M. This leads to the final expression:

$$Swaption_{6M}(T_{1Y}) \approx P_{OIS}(T_{1Y}, T_{2Y}) \cdot 2_{1Y,2Y} \cdot \left(F_{12M}(T_{1Y}, T_{1Y}) - K^*\right)^+$$

where the adjusted strike $K^*$ equals to

$$K^* = K + \Delta S_{1Y};$$
$$\Delta S_{1Y} = S_{12M}(T_{1Y}) - S_{6M}(T_{1Y})$$

Notice that we have arrived at an expression that can alternatively be seen as a 12M cap starting in 1Y with an adjusted strike. The adjustment to the strike is the mere difference in spreads. This implies that although a 12M cap is illiquid and hard to price, we can alternatively price it as a 6M swaption with an adjusted strike.

To emphasize this point, let us make the dependence on strike and volatility more explicit. We have shown that:

$$Swaption_{6M}(T_{1Y}, K) \approx Cap_{12M}(T_{1Y}, K + \Delta S_{1Y})$$

Or, equivalently:

$$Cap_{12M}(T_{1Y}, K) \approx Swaption_{6M}(T_{1Y}, K - \Delta S_{1Y})$$

We are hence able to price a 12M caplet with strike K, by simply pricing a 6M Euribor swaption with strike $K - \Delta S_{1Y}$.

Notice that the difference between the single- vs the dual-curve assumption is now translated to a difference in strikes. Thus it can be easily computed.

To summarise the findings so far we have the table below:

| **Relation between 12 Cap prices vs 6M swaption prices, as seen from time $T_{1Y}$.** | |
|---|---|
| **Single-curve assumption (D1)** | $Cap_{12M}(T_{1Y}, K) = Swaption_{6M}(T_{1Y}, K)$ |
| **Dual-curve assumption (D2)** | $Cap_{12M}(T_{1Y}, K) \approx Swaption_{6M}(T_{1Y}, K - \Delta S_{1Y}(T_{1Y}))$<br>$\Delta S_{1Y} = S_{12M}(T_{1Y}) - S_{6M}(T_{1Y})$ |

The above relations represent market prices as seen from time $T_{1Y}$. In order to convert them into mark-to-markets as seen at time $T_0$ we need to compute the discounted expectation value. The only remaining stochastic variables in the dual-curve relation (D2) that the expectation would act upon are the two spreads $S_{6M}(T_1)$ and $S_{12M}(T_1)$.

At this stage, we would in principle require a stochastic spread model e.g. the one proposed by Mercurio and Xie [1]. However, in order to arrive at an analytically tractable expression we will consider the simplest model for the spreads, namely one where they remain constant:

$S_{6M}(T_1) = S_{6M}(T_0)$ and $S_{12M}(T_1) = S_{12M}(T_0)$.

It has been suggested that this deterministic approach has been used by many banks[2], namely a model is chosen for the OIS curve and then forward curves are built at a deterministic spread over the OIS curve, thus assuming the martingale property in the formula above.

More elaborate models can be used (as described by Mercurio and Xie), which would introduce a volatility adjustment to the expression. We note that the order of the volatility adjustment would be small compared to $\Delta S$ .

With this assumption we can then write:

$$
\begin{aligned}
&Cap_{12M}(T_{1Y}, K) = \\
&= Swaption_{6M}(T_0, K - \Delta S_0) = \\
&= Annuity(T_0, T_{2Y}) \cdot Bachelier\left(R_{6M}(T_0, T_{1Y}, T_{2Y}), K - \Delta S_0, \pm_{6M}(K - \Delta S_0),^2{}_{T_0,T_{1Y}}\right)
\end{aligned}
$$

With

$$\Delta S_0 = S_{12M}(T_0) - S_{6M}(T_0)$$

and the annuity is the sumproduct of the two yearfractions and OIS zero-coupon bonds at the swap payments at 18M and 2Y).

Notice that due to the translation property of the Bachelier formula we can alternatively rewrite the last equation as

$$
\begin{aligned}
&Cap_{12M}(T_{1Y}, K) = \\
&= Swaption_{6M}(T_0, K - \Delta S_0) = \\
&= Annuity(T_0, T_{2Y}) \cdot Bachelier\left(R_{6M}(T_0, T_{1Y}, T_{2Y}) - \Delta S_0, K, \pm_{6M}(K - \Delta S_0),^2{}_{T_0,T_{1Y}}\right)
\end{aligned}
$$

In other words, the adjustment can be viewed as an adjustment to the strike or to the forward swap rate.

With either of the two formulas, the adjustment to the strike will also influence the volatility. The impact of this adjustment depends on the skewness and convexity of the smile in the neighborhood of the strike. Typically, the convexity is larger for short maturities. Hence we expect the impact to be larger in this area.

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[1] F. Mercurio and Z. Xie (December 2012) *Risk* 25 “The basis goes stochastic”, pp.78-83.
[2] F. Mercurio (2014) “New Innovations in Multi-Curve modelling & Discounting”, Global Derivatives Amsterdam May 2014.

# Comparison between strike-adjusted vs unadjusted approach

In some FSI practices, the methodology for the valuation of the 12M caps is based on:

- Valuing the undiscounted option using the Caplet Bachelier formula with:
  - The 12M forward rate as underlying and
  - The 6M volatility smile (as it is the only available).
- Discounting using the OIS zero-coupon bond.

Since the underlying 12M swap only consists of one payment, we have that the 12M forward rate is equal to the 12M forward swap rate and the OIS discount factor is equal the annuity of the underlying swap. The above computation is thus equivalent to a 12M swaption valuation using a 6M swaption vol. The above approach, however, neglects the strike adjustment that has been demonstrated in the previous section.

The level of discrepancy can be shown in the analysis below where we compare:

| **Pricing of 12M caps** | |
|---|---|
| **Without strike adjustment (E1)** | $Cap_{12M}\left(T_{1Y}, K\right) = Swaption\left(T_{1Y,} K, R_{12M}, \pm_{6M}\left(K\right)\right)$ |
| **With strike adjustment (E2)** | $Cap_{12M}\left(T_{1Y}, K\right) = Swaption\left(T_{1Y,} K - \Delta S_0, R_{6M}, \pm_{6M}\left(K - \Delta S_0\right)\right)$ <br> $\Delta S_0 = S_{12M}\left(T_0\right) - S_{6M}\left(T_0\right)$ |

where, for clarity, we have made explicit the dependence of the swaption on the forward swap rate and the volatility.

In the table below we show a numerical example of the discrepancy between the two methods. The data is based on EUR as of 29 Jan 2016. The numbers express the MTM in terms of normal (Bachelier) caplet vols.

| **WITH strike adjustment** | **Strike** | | |
|---|---|---|---|
| **Maturity** | 0% | 1% | 2% |
| **1Y** | 28.52 | 38.76 | 50.58 |
| **2Y** | 43.23 | 50.35 | 61.17 |
| **3Y** | 51.84 | 59.18 | 69.19 |

| **NO strike adjustment** | **Strike** | | |
|---|---|---|---|
| **Maturity** | 0% | 1% | 2% |
| **1Y** | 31.43 | 41.65 | 53.60 |
| **2Y** | 42.73 | 51.82 | 63.14 |
| **3Y** | 53.79 | 61.66 | 71.93 |

We see that although the two results are similar there is a systematic bias of the unadjusted approximation.

# How we can help

Our team of quants provides assistance at various levels of the pricing process, from training to design and implementation.

Deloitte’s team can be provide services both at Front Office level or for model validation and risk management.

Some examples of solutions tailored to your needs:

- A managed service where Deloitte provides independent valuations of vanilla interest rate products (caps, floors, swaptions, CMS) at your request
- Expert assistance with the design and implementation of your own pricing engine
- Stand-alone tools
- Training on market conventions, Bloomberg’s VCUB and SWPM conventions, stochastic pricing models, shifted SABR methodology, the volatility smile, or any other related topic tailored to your needs
- The Deloitte Valuation Services for the Financial Services Industry offers a wide range of services for pricing and validation of financial instruments

Why our clients haven chosen Deloitte for their Valuation Services:

- Tailored, flexible and pragmatic solutions
- Full transparency
- High quality documentation
- Healthy balance between speed and accuracy
- A team of experienced quantitative profiles
- Access to the large network of quants at Deloitte worldwide
- Fair pricing

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