Power law investing

Karel Volckaert
Senior Advisor
Econopolis

Outlier gains and losses are the overriding determinants of investment performance.

Proponents of classical finance dismiss these supernormal events as freak accidents and simply discard them from theory. They seem unaware that fat tails also have a decisive influence on the central properties—variance, correlation—of return distributions. We investigate some of the consequences for portfolio diversification and risk management.

If you invested 100 years ago in the Dow Jones Industrial Average and you missed the best 100 days, you would have actually ended up with less than what you started with[1]. If by uncanny foresight, you were able to avoid the ten worst days out of some 25,000 in that century, the terminal value of your wealth would have been three times as large as the return a passive investment in the Dow would have brought. The same findings essentially apply to any market or sample horizon. The impact of outliers on investment performance cannot be overestimated. Financial history truly is written by heroes—and villains.

About Econopolis

Econopolis (www.econopolis.be) is an unbiased, independent and global financial boutique. It employs a team of excellent, creative and well-informed professionals, who search for and develop socially responsible and sustainable investments, while relying on thorough research and analysis to safeguard and prepare assets for the new economic landscape and the next generation.

About the writer: Karel Volckaert

Karel Volckaert graduated in civil engineering (theoretical physics and control theory) at the Universiteit Gent. He has earned the right to use the chartered financial analyst (CFA Institute) and financial risk manager (Global Association of Risk Professionals) designations. After a stint as an academic researcher, Karel worked for a Belgian investment bank and an independent valuation specialist with a focus on intangible assets and financial instruments. He helps to establish the Econopolis macroeconomic views, leads the investment committee and the strategic committee, is fund manager and is responsible for portfolio risk analysis.

Heuristics rather than formulas

Mean-variance optimised portfolio theory, Black-Scholes option valuation and the capital asset pricing model all rely on the assumption of normally distributed price changes. Its proponents readily acknowledge that classical finance cannot accommodate these fat tails of reality: 19 October 1987 was a 21-sigma event with an unfathomably small probability of occurring—normally. Outliers are cast out in fact as 'acts of god' or as anecdotes that can be rationalised 'ex post'. What remains safely within the theory is nothing but white noise. Investors and risk managers alike need more realistic models.

Fifty years ago, Benoit Mandelbrot, the father of the fractal, introduced such models in finance.[2] Prices still (more or less) follow a random walk, but just not in regular clock time. The relevant time dimension is 'trading time', a warped version of equably flowing time in which price action will take place rather mildly on one day and yet quite wildly on the next. These models do account for the salient characteristics of real financial prices: persistent trends punctured by sharp spikes or breaks that tend to cluster. The distribution of price changes follows a power law in which the probability of a gradually larger move falls off much more slowly than in the normal distribution. As a result, a disproportional part of the expected payoff is concentrated in a very limited number of outcomes.

The mathematics need not concern us here. In fact, beyond structurally identifying power laws as a much more appropriate model for financial prices than the normal distribution, there is not much one can definitively say. Calibrating the parameters of those distributions from historical samples is a hazardous venture. The estimated probability of very large moves, albeit very small, can easily range over multiple orders of magnitude. Investors and risk managers must resort to resilient heuristics rather than rigorous formulas.

1 *Javier Estrada (2008), Black Swans and Market Timing: How Not to Generate Alpha, Journal of Investing (Fall), pp. 20-34. Estrada's findings are based on data up to the end of 2006. Returns account for capital gains but not for dividends*

2 *Benoit Mandelbrot (1963), The Variation of Certain Speculative Prices, Journal of Business 36, pp. 349-419. Cf. also Mandelbrot's student Eugene Fama (1965), The Behavior of Stock Market Prices, Journal of Business 38, pp. 34-105. For a recent popular account: Nassim Taleb (2007), The Black Swan: The Impact of the Highly Improbable, Random House*

Winner takes all

Fat tails imply that probabilities do not decline fast enough to ignore when it comes to extremely adverse outcomes. Rather than engaging in a futile attempt to predict time and place of catastrophe, investors must manage their exposure to such events. Airport security does not seek to identify terrorists beforehand; it aims to avoid a blow-up in mid-air. Nobody knows when a storm tide will flood a coastal city but it pays to construct levees.

Conversely, venture capitalists will mostly refrain from trying to pick the elusive winner. The bulk of their gains typically derives from a single investment whose return vastly outstrips the gains—and losses—of the rest. In the textbook's random walk, even the largest price increment contributes marginally to the overall dispersion in the sample. In a power law, the winner takes all, whether we consider the distribution of wealth in the population, bestseller rankings, or the '80/20' rule in customer sales contribution.

In order not to miss out on those 'tenbaggers', the value drivers in a well-diversified portfolio ought to significantly outnumber the positions advised in classical finance. We typically do not know the winner in advance. In fact, the frequency of being right becomes largely irrelevant. The (biggest) part is of the same order of magnitude as the sum-of-the-parts, which is just another way of saying investment returns exhibit fat tails.

The tunnelling effect

Contrary to what many investors think, price series with heavy tails appear quieter than normally distributed changes. Stocks will spend more time in a tunnel of, say, one standard deviation around the mean than the Gaussian 68.3% would predict. The low volatility masks the fact that when break-outs from the classically defined tunnel occur, they may well be very dramatic. History cannot tell us whether things can never happen.

Proponents of classical finance rely on time diversification to paper over the impact of outliers: on a sufficiently long horizon, risk is reduced through averaging and long-term investment performance in the limit is normally distributed. The presence of fat tails does not necessarily invalidate the law of large numbers but it will make the sample mean converge much slower towards its expected value. The 'large' in the law's name may be larger than most investors can tolerate or academics can find data for.

A similar disclaimer applies to the 'limit' of the central limit theorem. *"Das Unendliche ist nur eine façon de parler"*, Carl Friedrich Gauss famously protested when a friend considered a limit to have been achieved as soon as the conditions were met for a limiting process to occur. In most power laws, the probability of extraordinary price changes simply decays too slowly in the tail to be able to rely on the asymptotic Gaussian limit.

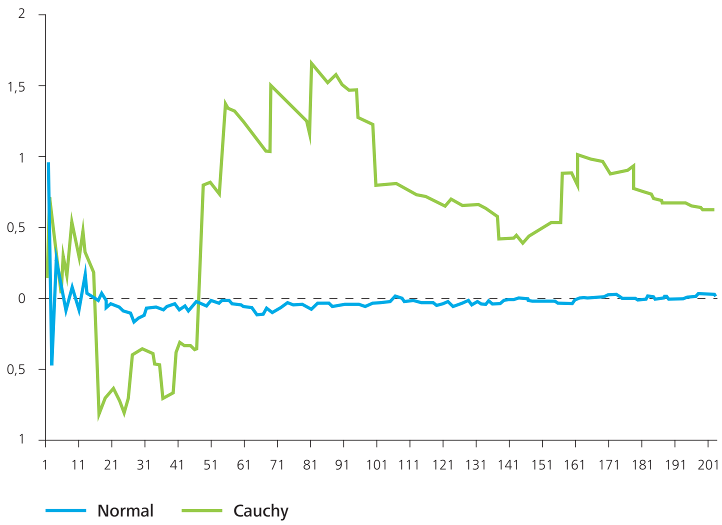
The tail wagging the dog

Outlier gains or losses matter for performance. Hunting down these 'black swans' is a wild-goose chase as we have seen. But arguably the most insidious and least understood effect of fat tails is their impact on the overall properties of the return distribution. Sample estimates such as variance or correlation that are central to classical finance, are critically affected to the extent that established portfolio management techniques break down in reality.

First, typical users of Markowitz portfolio optimisation regard the estimated rate of return and covariance as constants, which in itself is a fatal flaw. Furthermore, these estimates are quite sensitive to what happens in the tail of the distribution. Regression studies based on the method of the least squares may not replicate. Variance and correlation—in fact any measure making use of squares of return variables that exhibit fat tails—will differ from sample to sample[3]. In extreme cases, even the first-order sample mean is unstable.

Nobody knows when a storm tide will flood a coastal city but it pays to construct levees

3 Technically, power laws with exponents below two feature infinite variance



The graph shows the convergence, or lack thereof, of the mean of gradually larger samples. In blue, a normally distributed variable with a zero expected value; in green, a Cauchy distributed variable for which characteristically the expected value does not exist.

It is the absence of concentration in the classical model that justifies the use of standard deviation as a measure of volatility. When a substantial portion of overall sample variance derives from a small number of large price changes, the use of standard deviation to measure risk is problematic. And with sample averages doggedly refusing to converge to their expected values, diversification becomes debatable.

R.U.R.

Risk management cannot do away with the unpredictable and the unmeasurable. Its mission is to ensure that investment performance falls within the range of what the investor can reasonably expect. Fat tails make that task even more daunting but that ought not to deter us. The keywords in our view are recognition, understanding and reward.

Each individual position exposes the investor to a complex of value and risk drivers that recombine on the portfolio level. Risk management then consists of recognising those exposures at the security and the portfolio level, and understanding how each individual instrument contributes to the aggregate risk exposure. The reader will note we did not say 'identify' nor 'measure'. Conditional value-at-risk, extreme value theory, stress tests or even power law calibration, all venture into the realm of outliers where historical probabilities provide a flimsy foundation, and where recognising and understanding exposure should be the overriding consideration.

In the end, the trade-off to evaluate is whether the payoff profile is sufficiently rewarding for the investor to risk being exposed. Sometimes we take the leap, but the answer will be negative more often than follows from those pretty, polite techniques of classical finance.

To the **point**:

- Investment performance is overwhelmingly determined by one-day gains and losses which classical finance deems 'de facto' impossible
- Fat tails are not only important *per se*: they affect the central properties of return distributions too, to the extent that classical portfolio techniques break down in reality
- The probability of those outliers cannot be dependably determined from the limited historical samples we have
- Investors and risk managers must resort to resilient heuristics rather than rigorous formulas